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Evanescent modes are virtual photons

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Abstract. – Former QED-based studies of evanescent modes identified these with virtual photons. Recent experimental studies confirmed the resulting predictions about non-locality, non-observability, violation of the Einstein relation and the existence of a commutator of field operators between two space-like separated points. Relativistic causality thus is violated by the near-field phenomenon evanescent modes while primitive causality is untouched.

Introduction. – Electromagnetic waves can spread across classically forbidden regions in form of evanescent modes. These modes with a purely imaginary wave number k were originally thought of as a mathematical tool having no physical meaning. They matured in the last decades to a versatile tool in many areas of near-field optics and photonics [1].

QED-based studies of evanescent modes identified these with virtual photons [2–5]. This implies peculiar properties like non-locality and non-observability, violation of the Einstein relation and the existence of a non-trivial commutator of field-operators between space-like separated points. Recent experiments confirmed these theoretical results [6–8]. As a consequence, relativistic causality is violated by evanescent modes while primitive causality —cause precedes effect— is untouched as shown below.

To illustrate these remarks let us recall the properties of frustrated total internal reflection (short: FTIR) of double prisms. For an angle of incidence $\Theta > \Theta_c = \arcsin(1/n)$, geometrical optics predicts total reflection at the first prism-air interface, where n is the refractive index of the prisms. In the presence of a second prism a part of the beam tunnels across the air gap between the two prisms, *i.e.* across a region analogous to the square wall barrier of quantum mechanics (cf. fig. 1). Evanescent modes thus are the mathematical analogy of quantum mechanical tunneling, *i.e.* there are analogous solutions of the Helmholtz and the Schrödinger equations. The incoming beam actually travels some distance (D in fig. 1) parallel to the interface before being reflected and partially transmitted [9]. This effect is called Goos-Hänchen shift. The reflected and the transmitted components of the incoming beam have been measured to arrive simultaneously at the receivers in this symmetrical experimental design [9].

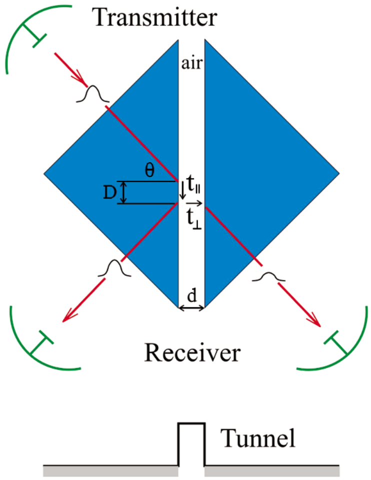

Fig. 1 – The prototypical set-up of measuring FTIR consists of two prisms, separated by a gap of air. This gap is for $\Theta > \Theta_c = \arcsin(1/n)$ a tunneling barrier for the incoming beam, where n is the prisms' refractive index. The incoming beam travels a distance D parallel to the interface before being transmitted and reflected. The symmetrical experimental set-up allows to determine the tunneling velocity as a sum of time delays for propagation parallel $t_{\|}$ and perpendicular $t_{\perp}$ to the prism-air interface.

The measured time delay $t_{\|}$ of 117 ps in the microwave experiment [9] for propagation parallel to the interface corresponds to the Goos-Hänchen shift. However, the time $t_{\perp}$ needed for the transmitted beam to cross the air gap is zero analogous to a prediction for electron tunneling [10, 11]. Results of zero tunneling time hold for all photonic experiments monitoring the fate of tunneling signals: the receivers detect a tunneled signal earlier than an airborne reference signal. The finite superluminal signal velocities measured are caused by a universal time delay at the entrance of barriers [12, 13] like the Goos-Hänchen shift in FTIR.

To elucidate the results of QED-based studies of evanescent modes we now recall the basic features of the classical wave velocities. Wave propagation in dispersive media is a far-field phenomenon described by velocity concepts derived in [14]. To match the universal speed limit of the special theory of relativity postulated for propagation in vacuum [15], some essential assumptions have been made in [14]: i) the medium was assumed to satisfy Lorentz-Lorenz dispersion and ii) a signal propagating in this medium was defined as a frequency band unlimited wave packet exhibiting a discontinuous wave front. This special frame then allowed to prove that the velocity of such a signal amounts at most to its front velocity bound by the speed of light in vacuum. The result is supposed to warrant relativistic causality. Experimental realizations of superluminal group velocities in regions of anomalous dispersion [16–18] match these causality-preserving criteria since simultaneously exhibiting pulse reshaping of the signal.

These velocity definitions, valid in case of optical media characterized by a complex refractive index, are not applicable in case of evanescent modes with a purely imaginary wave number.

Dispersion and causality. – The principle of causality can be phrased in macroscopic or microscopic terms [19]. The macroscopic formulations read: i) "a cause always has to precede an effect" (primitive principle of causality) or ii) "no signal can propagate faster than the speed of light in vacuum" (relativistic or Einstein causality). The microscopic principle of causality arising from quantum field theory states that "any commutator of field operators taken at two

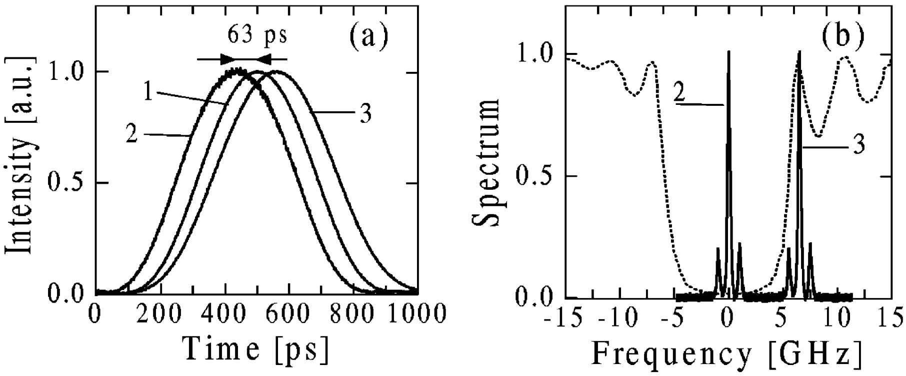

Fig. 2 – (a) Measured delay time of three digital signals and (b) shows a small region of the spectrum of the photonic lattice transmission with the forbidden gap (dotted line) [6]. The zero point in the mid of the forbidden frequency gap corresponds to the infrared signal frequency of $2 \cdot 10^{14}$ Hz. The forbidden gap is about 10 GHz broad. The signal spectra (solid lines) are normalized to largest amplitude. Pulse 1 was recorded in vacuum. Pulse 2 traversed the photonic lattice at superluminal velocity of $2c$. Since all of its frequency components are in the center of the forbidden gap of the lattice, the pulse components contain only evanescent modes. Pulse 3 was recorded for the pulse subluminal travelling through the fiber at frequencies outside the forbidden band gap. The tunneling barrier was a photonic lattice of a quarter wavelength periodic dielectric hetero-structure fiber.

space-like separated points has to vanish". The first two principles are not interchangeable as shown now.

The assumption of a Lorentz-Lorenz medium with a complex index of refraction satisfying the Kramers-Kronig relations [14] has been shown to be necessary and sufficient for relativistic causality to hold [19,20], *independent* of any special definition of a signal or its velocity. In the experimental studies of evanescent modes was, however, no Lorentz-Lorenz medium involved: the incoming signal always experienced a purely imaginary frequency-independent index of refraction in the relevant frequency band of the signal. The approach in ref. [14] thus is not appropriate to evanescent modes due to the imaginary wave number. As an example, the results of an experiment with digital signals are displayed in fig. 2. For pulse 2 all frequency components of the signal are in the forbidden frequency region satisfying the condition *sine qua non* for a superluminal signal velocity.

Different quantizations of evanescent modes have been presented in refs. [3, 4] showing i) that that evanescent modes have to be identified with virtual photons [3] and ii) that the commutator of field operators between two space-like separated points actually does not vanish thereby violating the microscopic causality condition [4]. This result originates apparently from the assumption of a constant real index of refraction for the frequency band of the incoming signal [4].

Frequency band limited signals. – In [14] a signal has been defined as a frequency-band–unlimited, but time-limited wave object. Such a mathematical signal does not match the restrictions on physical signals and physically feasible communication channels. The number of telegraph signals to be transmitted over a line has been shown to be proportional to the transmission time and bandwidth [21]: any signal has to satisfy the basic uncertainty relation of an arbitrary wave packet reading

$$\Delta\nu \cdot \Delta t \geq 1,$$

where $\Delta\nu$ is the frequency bandwidth of the signal and Δt its duration. The time-bandwidth product of a signal thus requires a finite frequency band and a finite duration thus violating the assumptions of [14].

The uncertainty relation and the sampling condition imposing a finite frequency band to guarantee a faithful transmission of signals show the impossibility of submitting a frequency-band–unlimited signal over a physical communication channel [22]. A front velocity is of limited meaning [23]. This shows, once more, that the derivation of Einstein causality in [14], based on an unrealizable notion of a signal, bears no impact on the existence of superluminal signals in the near-field, *i.e.* on tunneling of frequency-band–limited signals. Incidentally, according to Planck's quantization of radiation, a frequency-band–unlimited signal implies an infinite signal energy since the energy of any frequency component ν is $h\nu$ [7].

Evanescent modes are not observable. – Evanescent modes (and tunneling particles) are not observable inside a barrier [24, 25]. They do not interact with an antenna as long as the system is not perturbed thereby transforming an evanescent mode back into a propagating electromagnetic wave. Evanescent modes display moreover some outstanding properties compared to far field propagation:

1) Evanescent modes with an imaginary wave number k violate the Einstein relation, *i.e.* $W^2 \neq (\hbar k)^2 c^2$ [3], where W, k and c are the total energy, the wave number and the velocity of light in vacuum.

2) An evanescent field does not interact with real fields due to the refractive index mismatch. Fields can only transmit energy if for the reflection $R < 1$ holds. If n_1 represents the imaginary refractive index of an evanescent region and n_2 represents the refractive index of the dielectric medium representing a receiver then the square of the absolute value
$$R = |r|^2 = \frac{|n_2 - n_1|^2}{|n_2 + n_1|^2}$$
equals 1 and total reflection takes place. This result is similar to that of free charge carriers. Below the plasma frequency, for instance, radio waves are totally reflected by the ionosphere.

3) In order to observe a particle in the exponential tail of tunneling propability, it must be localized within a distance of order of $\Delta x \approx 1/\kappa = 1/ik$. Hence, its momentum Δp must be uncertain by
$$\Delta p > \hbar/\Delta x \approx \hbar\kappa = \sqrt{2m(U_0 - W_{kin})}.$$
The particle of energy W_{kin} can thus be located in the nonclassical region only if it is given an energy $\Delta W = U_0 - W_{kin}$ thus raising it into the classically allowed region [25]. For evanescent modes in case of double prisms, for instance, the argument of the square root takes the form $k_0^2(n_2^2 \sin^2(\theta) - n_1^2)$, where $k_0 = \omega/c, n_1$ and n_2 are the refractive indices of the gap and of the prisms, respectively.

4) Non-locality takes place as $t(T) = t(R)$ holds, *i.e.* the barrier reflection and the transmission times of signals are equal [7, 8].

The limit of the near-field: Johnson-Nyquist noise. – For a unique identification of an incoming signal it is mandatory that its power is above the inherent thermal noise level of the receiver. In case of photonic tunneling, this condition requires the power of a tunneled signal

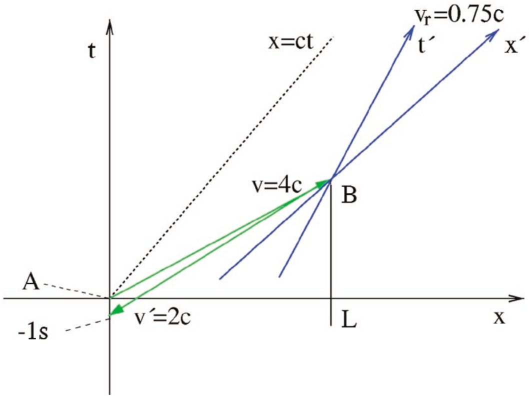

Fig. 3

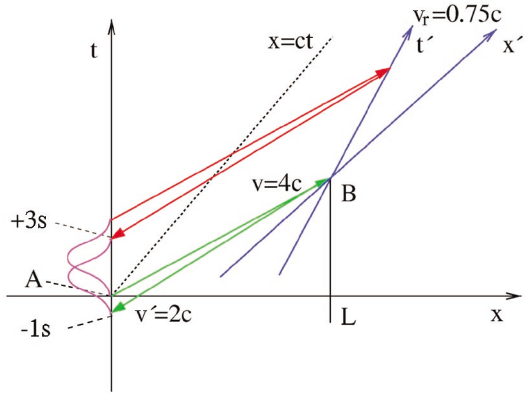

Fig. 4

Fig. 3 – A (0,0) and B with O (x, t) and O$'$ (x', t') moving with a relative velocity of $0.75c$. The distance L between A and B is 2000000 km. A makes use of a signal velocity $v_S = 4c$ and B makes use of $v'_S = 2c$ (in the sketch is $v \sim v_S$). The numbers in the example are chosen arbitrarily. The signal returns non-causal, -1 s, in the past of A.

Fig. 4 – In contrast to fig. 3 the pulse-like signal has now a finite duration of 4 s. This data is used for a clear demonstration of the effect. In all superluminal experiments, the signal length is long compared with the measured negative time shift. In this sketch the signal envelope ends in the future with 3 s.

to be above the Johnson-Nyquist noise P_N of the detector [8]. This thermal noise (cf. [26]) is generated by the spontaneous thermal fluctuations of voltage across an electric circuit element. The expression for thermal noise power generated by a resistor in thermal equilibrium reads

$$P_N = kT\Delta\nu,$$

where k is the Boltzmann constant, T is the temperature, and $\Delta\nu$ the frequency range of the signal. This relationship allows a general estimate of the extension of the near-field bearing superluminal signals: this range is limited by the condition

$$P(x) = P_0 e^{-2\kappa x} \geq kT\Delta\nu,$$

where P_0 is the incident power of the evanescent mode and κ is the imaginary wave number of the superluminal signal in the near-field. This condition limits the near-field of evanescent modes up to the order of some thousands of wavelengths depending on the transmitter's power.

Information velocity: from cause to effect. – A physical signal begins gradually and ends gradually, having no sudden start and no sudden end. It has often been argued, however, that the information content of a signal is encoded in some kind of discontinuity like a sharp front. This discontinuity can be shown to travel with (sub)luminal speed across the medium thereby rescuing Einstein causality [16–18, 27]. This line of arguments ignores the physics of communications:

i) any discontinuity in a signal, when sampled, leads to aliasing effects and cannot uniquely be reconstructed at the receiving end [22];

ii) the assumption of information encoding in discontinuities is not realistic:

- It is convention in many systems to encode information in the half-width of signals [28]. Since the half-width of signals is preserved in photonic tunnelling despite exponential damping of the signal [7, 8], these signals did travel with superluminal speed and were detected earlier than airborne ones;
- Mobile phone standards (like GSM or UMTS) encode the information in the phase of the signal. Since the phase of an incoming signal remains unchanged when tunneling across a forbidden region, a calculation of the information velocity via the phase-time [29] again yields a superluminal information velocity.

Primitive causality is preserved. – Superluminal signals preserve the primitive principle of causality. To demonstrate this fact we first recall the classical arguments linking superluminal signals to manipulations of the past (cf. fig. 3). Let lottery numbers be presented as points on the time coordinate with zero time duration. At $t = 0\,\mathrm{s}$ the counters are closed. A sends the lottery numbers to B with a signal velocity of $4c$. B, moving in the second inertial system at a relative speed of $0.75c$, sends the numbers back at a speed of $2c$, to arrive in the first system A at $t = -1\,\mathrm{s}$, *i.e.* just in time to deliver the correct lottery numbers before the counters close at $t = 0\,\mathrm{s}$.

The time shift of a point on the time axis of reference system A into the past is given by [30]

$$t_\mathrm{A} = -\frac{L}{c} \cdot \frac{v_r - c^2/v_S - c^2/v'_S + c^2 v_r/(v_S \cdot v'_S)}{c - c\, v_r/v'_S},$$

where L is the transmission distance of the signal and v_r is the velocity between the two inertial systems A and B. The condition for the change of chronological order is that the time shift between the systems A and B is $t_\mathrm{A} < 0$. We now take the finite duration of physical signals —like the pulses sketched along the time axis in fig. 4— into account.

Assuming a signal duration of $4\,\mathrm{s}$, the complete information is obtained with superluminal velocity at $+3\,\mathrm{s}$ (cf. fig. 4). The finite duration of signals is the reason why a superluminal velocity does not violate the principle of causality even if the signal travels with $v_S \gg c$. The dispersion in the transmission of tunneling barriers enforces a narrow frequency band of a signal and a long duration in order to suppress signal reshaping. As a consequence, an increase of v_S or v'_S cannot violate the principle of causality as shown in [7].

Summary. – Evanescent modes have been shown to violate relativistic causality and the microscopic causality condition while leaving the primitive principle of causality untouched. All the special properties of virtual photons elaborated by QED approaches have been observed in several experiments with evanescent modes.

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